MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 102 No. 6

MEETING OF 1942 OCTOBER 9

Professor S. Chapman, President, in the Chair

The election by the Council of the following Fellows was duly confirmed:—

Captain William Muir Ferguson Macfarlane, Fleetwood Navigation School, Fleetwood, Lancashire; and

Walter C. Rand, 58 Woodhouse Road, North Finchley, London, N. 12.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Kenneth Charles Blackwell, Royal Observatory (Greenwich), Cornerways, Abinger Hammer, near Dorking, Surrey (proposed by H. Spencer Jones);

Frederick Richard Cripps, 6 Belmont Road, Bushey, Watford, Herts (proposed by J. G. Porter);

Colin Alistair Ronan, 6 College Road, Ealing, London, W. 13 (proposed by Alan J. Skinner); and

Cyril Arnold Williams, M.A., Monkton House School, Cardiff, S. Wales (proposed by A. D. Thackeray).

Twenty presents were announced as having been received since the last ordinary meeting, including:—

V. Hase: *Peculiar Changes in the Shell Spectrum of γ Cassiopeiæ (presented by the author);

H.M. Nautical Almanac Office: Apparent Places of Fundamental Stars, 1943 (presented by H.M. Stationery Office).

The President welcomed Professor Fournier and Professor Koyré, Fighting French scientists, and other visitors representing various learned Societies and Institutions, and announced that the meeting was being held to commemorate the three astronomers, Galileo, Newton and Halley. The President first called upon Mrs. John Evershed to give brief histories of the three men, and then invited Professor Dingle, Professor

Plummer, and the Astronomer Royal, Dr. Spencer Jones, to give accounts of the scientific achievements of Galileo, Newton and Halley respectively. Dr. Spencer Jones closed his address with slides of portraits of Halley.

At a Council meeting held 1942 December 11, the following were elected Fellows of the Society:—

Charles Going Ambrose, M.D., c/o Lloyds Bank Ltd., 27 Whitechapel High Street, London, E. 1;

Edward James Cope, Mendip, West Malvern, Worcestershire;

Robert James Porter, B.Sc., 22 Marlborough Avenue, Hessle, East Yorkshire; and

P. A. Scott-Iversen, 11A Newbold Terrace, Leamington Spa, Warwickshire.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

John Thornton Burge, Glenhurst, Braeside Road, West Moors, Dorset (proposed by Maurice A. Ainslie);

Ben Burrell, 93 Woodhouse Road, Doncaster, Yorkshire (proposed by Robert Barker);

Henry J. Green, M.I.C.E., M.I.M.E., Corvette, Church Lane, Pinner, Middlesex (proposed by Ernest G. Wainwright);

Aylmer Joseph Probyn Maude, 3 Airlie Gardens, London, W. 8 (proposed by W. MacIntyre);

Frank Peverett, 7 Grandage Terrace, Whetley Hill, Bradford, Yorkshire (proposed by James F. S. Shepherd);

John Benson Sidgwick, Greenfields, Llanfair Road, Newtown, Mid Wales (proposed by H. H. Plaskett);

Henry Bertram Tetlow, 2 Lymm Hall, Lymm, Cheshire (proposed by William Porthouse); and

Vladimir Vand, D.Sc., 32 Temple Fortune Hill, London, N.W. 11 (proposed by George Alter).

ORBITAL MOTION AND MEASUREMENTS FOR PARALLAX

J. Jackson, F.R.S.

(Received 1942 December 14)

In the analysis of the measures of parallax plates three unknowns are introduced into the equations of condition, viz. a constant, the proper motion and the parallax. When the observations extend over 5 or more epochs of maximum parallactic displacement the unknowns are very well separated. Extension beyond 5 epochs does not greatly improve the separation of the unknowns, but the longer interval between the first and last plates increases the accuracy of the proper motion and additional epochs reduce the influence of errors which may be systematic at an epoch.

It is of course realised that the star under observation may have an orbital motion of its own which would invalidate this simple method of reduction. In the case of visual binaries such as α Centauri and *Sirius* where the orbital motion (including the relative masses) is known this can be allowed for without the introduction of further unknowns. Cases where this is necessary, however, are not very numerous.

Observations made at 5 epochs for three unknowns frequently show residuals which are systematic, though not greater than might be expected from accidental error. It is very tempting to assume that these residuals arise from orbital motion, but there is great difficulty in discovering true orbital motion from parallax measurements. At a minimum it is necessary to introduce at least three additional unknowns into the equations of condition, one for the period, one for the phase and one for the amplitude. Of these the period is usually determined graphically: after which the equations are solved for the other five unknowns. If this is done with only 5 epochs it is virtually equivalent to solving 5 equations for six unknowns and the orbit deduced is wholly unreliable.

Let us examine how it is done. Suppose that our observations are made at ideal times. If 2μ is the annual proper motion and π the parallax, the equations of condition are:

$$c-2\mu-\pi=m_1, \qquad c-\mu+\pi=m_2, \qquad c-\pi=m_3, \qquad c+\mu+\pi=m_4,$$
$$c+2\mu-\pi=m_5.$$

Suppose $m_1=1$, $m_2=m_3=m_4=m_5=0$. If we form the normal equations in the usual way and solve them, we get $c=\frac{1}{6}$, $\mu=-\frac{1}{5}$, $\pi=-\frac{1}{6}$, and the residuals are $+\frac{4}{15}$, $-\frac{1}{5}$, $-\frac{1}{3}$, $+\frac{1}{5}$, $+\frac{1}{15}$. These can be fitted into a curve with a minimum period of just under two years.

If $m_2=1$ and the other m's are zero, we get $c=\frac{1}{4}$, $\mu=-\frac{1}{10}$, $\pi=\frac{1}{4}$, and the residuals are $-\frac{1}{5}$, $+\frac{2}{5}$, 0, $-\frac{2}{5}$, $+\frac{1}{5}$. These residuals fit into a curve with a period of just under two years.

If $m_3=1$ and the other m's are zero, we get $c=\frac{1}{6}$, $\mu=0$, $\pi=-\frac{1}{6}$, and the residuals are $-\frac{1}{3}$, 0, $+\frac{2}{3}$, 0, $-\frac{1}{3}$. These residuals again fit into a curve with a minimum period of two years.

Similar results follow for the other two special cases. So for all five cases we find that the residuals can be fitted into a curve with a period of about two years. As any case can be considered as the weighted mean of the five special cases, we see that the residuals for any values of $m_1 \ldots m_5$ will yield a minimum period of just under two years.

This result can be seen without considering the special cases. The residuals for the second and fourth equations must be equal and opposite: those from the three others must balance and show the opposite proper motion to that derived from the other two.

If the residuals for the second and fourth epochs are largest the case is similar to $m_2 = 1$ above. If the other residuals are larger the case is similar to $m_3 = 1$. If they are all equally large the case is similar to $m_1 = 1$. The minimum period of about $1\frac{3}{4}$ years corresponds to $m_2 = 1$.

Now consider the results from observations extending over 7 epochs. We have the two additional equations of condition:

$$c - 3\mu + \pi = m_0, \qquad c + 3\mu + \pi = m_6.$$

The residuals for the special cases are given below, it being considered that all the m's, except the one in the heading, are zero, that one being given an integral value to avoid the use of fractions in the residuals.

	$m_0 = 14$	$m_1 = 42$	$m_2 = 14$	$m_3 = 3$
r_0	+6	− 9	− 5	0
r_1	−3	+22	− 1	−1
r_2	−5	− 3	+10	0
r_3	0	−14	0	+2
r_4	−2	+ 3	− 3	0
r_5	+3	− 8	+ 1	−1
r_6	+1	+ 9	− 2	0

These sets of residuals can be roughly fitted into sine curves indicating periods of about $2\frac{1}{2}$, $1\frac{1}{2}$, $1\frac{3}{4}$, 2 years. It again appears that residuals will naturally fit a curve with a period of about two years. In fact this could be expected from general considerations; a large residual at any epoch will be associated with residuals of the opposite sign a year earlier and a year later. This argument fails when the largest residuals approach an end of the series.

In *Lund Medd.*, Series 2, No. 92, 1938, Holmberg has investigated residuals from parallax observations. He states that for 25 per cent. of 240 stars common to the Allegheny and McCormick lists periodic displacements were found. He does not state the periods, but for 8 stars investigated more closely he gives periods of 1·44, 2·99, 1·82, 2·47, 1·61, 1·59, 1·88, 1·55, 1·80 years, the last two periods being attributed to two invisible companions of 61 Cygni B.

Holmberg's results do not, however, rest on single series of observations such as considered above. For *Prox. Centauri* he uses two series of observations, for the other stars three or more. In the case of *Prox. Centauri* the first set of 5 residuals fits closely into a curve of the form $m_3 = 1$ for 5 epochs shown above, and the second set for 6 epochs into a similar curve. Both sets of residuals fit better into curves with a period of 2 years than Holmberg's 2·47 years; the longer period is required to fit the two sets of residuals together. It may be stated here that two series of observations of this star covering 11 epochs made at the Cape Observatory do not support the periodic motion derived by Holmberg.

For the other stars discussed by Holmberg there are more extended observations. An examination of the curves shows that the fits are not so good when the series overlap considerably. It may be pointed out that with merely accidental distribution of the residuals it will be found that phases will agree within 45° for 25 per cent. of the cases, so that very strong evidence is required before periods of the order of 2 years can be confirmed. Two further objections can be urged against the immediate acceptance of Holmberg's results. The first is that the published residuals do not appear to have been analysed for the three usual unknowns along with the new ones. This is essential with periods and amplitudes such as those derived by Holmberg. It is surely remarkable that for the periodic motion investigated by Holmberg in which the period and the phase have both to be determined, the probable error of the amplitude is more often than not smaller than that of the parallax for which the period and phase are known

beforehand. The second objection is that the derived amplitudes do not show any connection with the parallaxes while rough proportionality might have been expected. Thus for *Prox. Centauri* with a parallax of $0''\cdot779 \pm ''\cdot008$ the periodic change has an amplitude of $0''\cdot010 \pm ''\cdot003$, while for γ Andr BC the figures are $0''\cdot015 \pm ''\cdot003$ and $0''\cdot022 \pm ''\cdot004$ respectively. The small periodic change found for *Prox. Centauri* may be taken as indicating that this star has no companion more than twice as massive as Jupiter.

In *P.A.S.P.*, 54, 137, 1942, Miss Vinter Hansen draws attention to stars for which the existence of orbital motion is known from spectroscopic observations, and gives a list of those for which $a \sin i$ exceeds 0·4 of an astronomical unit. For such stars a displacement comparable with at least half the parallax may be expected, and with a known period and eccentricity only two unknowns will be required in each co-ordinate to represent the observations accurately. Moreover, as $a \sin i$ is given in kilometres by the spectroscopic observations, while a is given in arc by the parallax plates and i is also determined, an independent absolute determination of the parallax can be derived provided the companion star does not affect the position of the image on the plates. Very valuable work of this nature has already been done, especially by Dr. Alden in a series of papers published in the *Astronomical Journal* beginning as long ago as 1925.

Miss Vinter Hansen remarks that parallax determinations may be seriously affected by orbital motion. This effect can be easily investigated. With ideal distribution for parallax we get, with orbital motion, equations of the form

$$\begin{array}{llll} c-2\mu-\pi=-a\sin 2\theta+b\cos 2\theta, & & c-\pi=b, & c+\mu\ +\pi=a\sin\theta\ +b\cos\theta, \\ c-\mu\ +\pi=-a\sin\theta\ +b\cos\theta, & & & c+2\mu+\pi=a\sin 2\theta+b\cos 2\theta, \end{array}$$

where 2θ is the orbital motion in a year.

If we form the normal equations in the usual way and solve we get

$$6\pi=b(-1+3\cos\theta-2\cos 2\theta),$$

or with 7 and 9 epochs we get

$$\begin{aligned} 12\pi&=b(-2+3\cos\theta-4\cos 2\theta+3\cos 3\theta), \\ 20\pi&=b(-2+5\cos\theta-4\cos 2\theta+5\cos 3\theta-4\cos 4\theta). \end{aligned}$$

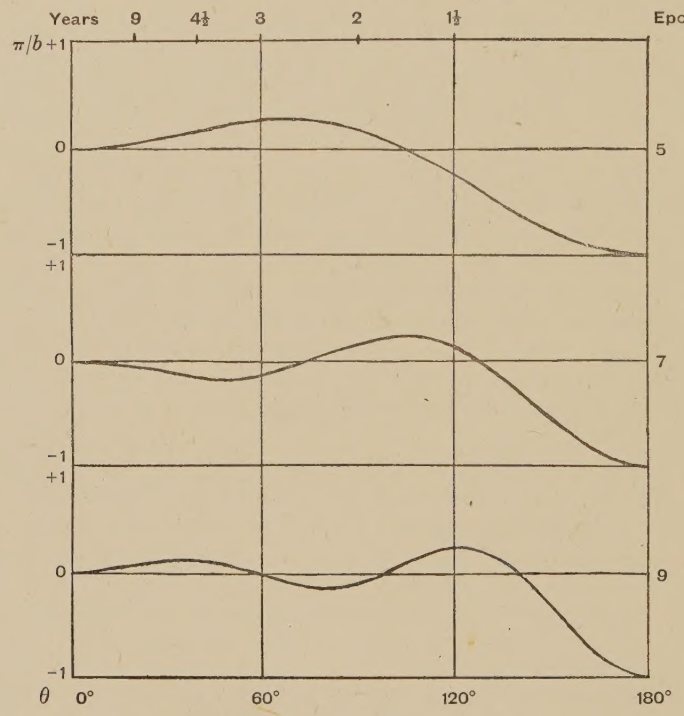
The diagram shows the values of π/b for values of θ from 0° to 180°, *i.e.* for periods greater than 1 year.

As is obvious without any calculation we find $\pi=-b$ when the orbital motion has a period of 1 year. With θ less than 120°, *i.e.* for a period greater than $1\frac{1}{2}$ years, the maximum value of π is $\pm\ 0{\cdot}26b$. For small values of θ the error introduced into the parallax is proportional to $b\theta^2$, and as Kepler's laws give $b\theta^{2/3}$ as a function of the masses the error will be less for long periods than short ones. This argument would have to be considerably modified if the epochs of observation departed greatly from the series considered. For example, if observations were made at intervals of $1\frac{1}{2}$ years instead of $\frac{1}{2}$ year the effect of orbital motion would be greater. However, most series of observations extend over consecutive epochs of maximum parallactic displacement and the argument given has fairly general application.

Miss Vinter Hansen's list contains 61 spectroscopic binaries. Twenty-three have periods under 2 years, and two of these, viz. ξ U Maj A and χ Drac, with large parallaxes may have errors as large as $''\cdot010$ introduced through neglect of orbital motion. It is unlikely that any of the others are seriously affected, but measures may well be examined for a few others with a parallax somewhat in excess of $0''\cdot02$. For the 38 with periods over 2 years there are only five for which the error is at all likely to exceed $''\cdot005$, and only two, viz. β Capr and ν Octn, might reach $''\cdot01$. Of the four stars specially noted three have been analysed for orbital motion by Alden and the fourth by Stearns.

As orbital motion is as likely to increase as to decrease the parallax we need not fear any serious effects on parallaxes in general.

This does not mean that the discussion of parallax observations for orbital motion is useless. I think that all stars with a large parallax, say 0″·100 or over, might be given extended series of observations with a view to the detection of invisible companions. It would be interesting to find companions to the nearest stars: it should be possible



Values of π/b for different values of θ when the observations are spread over 5, 7 and 9 consecutive epochs.

to find, if they exist, bodies not greatly more massive than Jupiter. Also spectroscopic binaries with large values of $a \sin i$ should be measured for orbital motion when displacements as large as ″·02 may be expected. As already noted, they are valuable in giving in conjunction with spectroscopic observations an independent determination of the parallax, provided that the second star may be considered as non-luminous. About 30 stars of Miss Vinter Hansen's list belong to this class, and the most important have already been discussed by Alden and others.

Summary

The influence of periodic motion in stars measured for parallax is examined. It is found that the residuals of parallax measurement tend to show spurious periods of about 2 years. While orbital motion with a period of about 1 year may make parallax determinations unreliable, the parallax will not be seriously affected if the period is longer than 1½ years. Extended series of parallax observations should be made of the nearest stars and of stars with large values of $a \sin i$ and reasonably large parallax.

Royal Observatory, Cape:
1942 *November* 11.

OBSCURATION AROUND THE NORTH POLE

E. G. Martin

(Communicated by the Astronomer Royal)

(Received 1942 December 5)

A previous paper * gave an account of the results found from an examination of counts of stars in the Greenwich Astrographic Zone. These counts were not compiled for the region within 3° of the North Pole of the sky, but as it is now fairly certain that there is obscuration in this area it seemed desirable to examine the region. On examining the work of other investigators on this subject, it is clear that they are not discussing the same thing. For example, Seares † speaks of the area north of Dec. +80° as being covered by a veil of obscuration, while Shapley and Jones ‡ give some details of an irregular obscuration, approximately two degrees in diameter at Dec. +87°·3. They suggest a distance of 200 parsecs from the Sun. The same authors § give an account of a survey of 16,639 galaxies north of Dec. +70°, from which they find evidence for a number of obscured regions. One of these is at Dec. +86°·5; the central portion of this region is stated to be "heavily obscured by a dark cloud that appears to be some five or six hundred parsecs distant." To this statement is added a footnote: "Erroneously given as 200 parsecs in *H.B.* 905." Stebbins, Huffer and Whitford || discuss the colours of A-type stars near the North Pole; using all Ao stars north of Dec. +80° they conclude that there is obscuration over the whole area, and it is suggested that this takes place at a distance of between 200 and 300 parsecs. W. Becker ¶ finds obscuration appearing sharply at 270 parsecs. Recently, Keenan and Babcock ** find absorption beginning at 200 parsecs, although by reclassification of the spectral types they find that the total absorption is less than that obtained by Stebbins.

It is therefore clear that there is evidence of a general veiling over a region 10° in radius around the North Pole, and of a heavily obscured area at Dec. +87°; the material causing the absorption in the two cases is probably at considerably different distances. The recent publication of the "Magnitudes and Colours of Stars North of +80°" by Seares, Ross and Joyner provides material for an examination of this portion of the sky, though it is regrettable that the spectral types of most of these stars have not been published.

To investigate absorption, good measures of colour, spectral types and distances are needed; the colours and spectral types must be independently determined. The distances will always be subject to considerable uncertainties, which greatly exceed the uncertainties of the colour determinations. Counts of stars will also provide material for the investigation of obscured areas, but the counts must extend to stars fainter by several magnitudes than the limiting magnitude which it is desired to discuss. For example, the magnitudes of the stars in the Greenwich Astrographic Catalogue within 3° of the North Pole extend to the 14th magnitude, but are complete only to magnitude 13. Owing to the spread in the luminosity function of the stars it would

* *M.N.*, 102, 237, 1942.
† *Mt. Wilson Comm.*, 119, 1936.
‡ *H.B.*, 905, 1937.
§ *H.A.*, 106, 1, 1938.
|| *Ap.J.*, 90, 227, 1939.
¶ *Zs. f. Ap.*, 17, 292, 1939.
** *Ap.J.*, 93, 64, 1941.

be necessary for the counts to be complete to the 16th or 17th magnitude to be sure of the density distribution at the 13th magnitude. Therefore complete counts to the 13th magnitude only ensure knowledge of the distribution to the 9th or 10th magnitudes and these stars would hardly be at a distance large enough for the purpose.

The Greenwich catalogue is complete to a fainter magnitude limit than any other covering the polar region. To complete the discussion of the Greenwich Zone, the region within 3° of the Pole has been examined. The area was divided into zones, one degree wide, zone +89° being subdivided into four quadrants each 0·8 square degrees in area, and zones +88° and +87° into twelve segments each, 0·8 and 1·3 square degrees in area respectively. The number of stars were then counted for each half-magnitude from $9^m{\cdot}0$ to $13^m{\cdot}0$, below which magnitude the catalogue is not complete. The comparison with *Gron. Pub.*, 43, Table 10, gave an average difference of +0·17 in log N_m compared with +0·20 for the region +64° to +86°. The p.e. in log N_m is ±·137 as against ±·102 in the zones of lower declination above 25° galactic latitude. No region within the 3° area appears to be outstanding as judged by the criterion of the previous paper, viz. having a residual of more than three times the p.e. A careful scrutiny of the numbers has been made to endeavour to detect whether any particular region has begun to feel the effect of an absorption as shown by a falling off in the number of the faintest stars. At $13^m{\cdot}0$ there is a definite suggestion that the region at 11^h, +87° is deficient in stars, but at $12^m{\cdot}0$ the deficiency is doubtful. In the previous paper, the region at 3^h, +86° had a negative residual just large enough to deem the area outstanding.

It would therefore appear from these counts unlikely that stars of the 8th magnitude are obscured, or that there exists an absorption at a distance of 200 parsecs. The counts suggest, in one region, a falling off in the numbers of stars at the 13th magnitude which if real would place the absorption at a much greater distance. It is well to study the diagram and the exact wording of Shapley and Jones.* Their plate covered 21 square degrees and reached magnitude 17·7. They state that: "The obscuration is easily recognised upon inspection of this long exposure plate. It is not readily evident for shorter exposures with the same telescope or for patrol plates of one-fourth the scale or less." To be "easily recognised" by inspection, the object must be considerably above the threshold density of the plate, say three to five magnitudes. This would mean that stars of the 12th to 14th magnitudes were affected. There are many older plates available which reach to the 13th magnitude, and if an obscuration effecting 8th magnitude stars existed, it would undoubtedly have been noticed much earlier, especially as the region round the North Pole has been extensively studied. It seems therefore improbable that 8th magnitude stars are strongly obscured; very accurate determinations would be needed to detect a possible effect of slight obscuration from star counts.

As the Greenwich star counts gave no definite support to the hypothesis of an obscuration at a small distance from the Sun, other lines of investigation were tried. For this purpose Petersson's † study of stars in the Carrington Zone was used. Petersson derived distances of these stars from spectrophotometric measures of the break in the continuous spectrum at the G band, together with measures of the cyanogen absorption. These measures afford a means of determining the absolute magnitudes and hence the distances of the stars. The same catalogue gives determinations of spectral types on the Upsala classification derived from the same plates. The colours of the A-type stars were extracted from the Mt. Wilson Catalogue and were tabulated against Petersson's distances. Stars were grouped according to distances of 0–150, 151–200, 201–300, etc. parsecs. The first group with distances from 0 to 150 parsecs is probably and must in any case be assumed to be free from absorption. The colours of the stars in this group are therefore deemed to be normal colours, affording the means to derive colour

* *H.B.*, 905, 1937.
† *Medd. Astr. Obs. Upsala*, 29, 1927.

excesses for obscured stars. Stars of types Ao and A2 only were used, the mean colour index of the 8 Ao stars being −0·07. The following results emerge:—

Distance (parsecs)	95	183	252	350	432	547
C.E. (magnitudes)	0·00	−0·01	0·00	+0·15	+0·14	+0·23
No. of stars	17	13	12	13	8	5

Thus there is no evidence of absorption up to a distance of 250 parsecs. Beyond 300 parsecs some absorption appears to be present, but the determination is not very certain because of the small number of stars available.

If the colours of the A-type stars are tabulated against the distances used above, using the mean of the spectral types derived at Harvard, Yerkes and Upsala instead of those derived at Upsala alone, the colour of the 10 normal Ao stars is −0·05 and the groups, including types Ao, A2 and A3, give the following results:—

Distance (parsecs)	101	182	222	270	410
C.E. (magnitudes)	−0·01	−0·03	−0·09	−0·10	−0·04
No. of stars	23	15	9	11	3

There would appear to be no evidence of absorption from these figures. If we select the A-type stars from the mean types derived at Harvard and Yerkes north of Dec. +80° from the list by Keenan and Babcock, using Yerkes distances but Mount Wilson colours, we find:—

Distance (parsecs)	113	187	250	335	427
C.E. (magnitudes)	−0·04	−0·05	−0·05	−0·09	−0·02
No. of stars	9	10	16	9	2

The determination is weak, but there is again no definite suggestion of any reddening.

The data given by W. Becker for the Selected Area No. 1, consisting mainly of faint F-type stars of photographic magnitudes 9 to 13, were examined. They cover a range of distances between 150 and 750 parsecs. It may be noticed that nearly all the strongly positive excesses relate to the faintest stars and are marked as of doubtful quality by the author; the spectral types of these stars also suffer from this defect. By altering the grouping in distance, it was found that the graph of colour excess against distance was modified, and Becker's absorption at 270 parsecs was so smoothed that the distance of its commencement in space was very difficult to determine.

Mean Colour Index of Ao Stars.—The data used above have also yielded mean values of the colour index of the average Ao star, these being $-0^m.07$ or $-0^m.05$ according to the particular classification used. It has, however, been suggested that for unobscured Ao stars the value of the colour index is $-0^m.14$. Indeed, Seares states definitely in *M.W. Comm.*, 119, 1936, that the mean colour index of Ao stars on the international scale is not zero but $-0^m.14$, with an uncertainty of $0^m.02$ or $0^m.03$. More recently Seares * gives four references to investigations which result in a mean colour index of $-0^m.14$ for unobscured Ao stars. This figure has also been used to correct the zero of the Harvard photovisual magnitudes, and the following relation between international and Harvard photovisual magnitudes is given:—

$$IPv - HPv = +0.10 + 0.06c.\dagger$$

It is stated that the difference of zero point is evidently the difference in colour index between the average Ao stars at Dec. +15° and the slightly reddened Ao stars of the North Polar Sequence.

The Ao stars have therefore been carefully examined to see whether it is possible to derive from them a mean colour index of $-0^m.14$. All the stars classified at Harvard as of type Ao, in the Mount Wilson Catalogue of magnitudes and colours of stars north

* *Trans. I.A.U.*, 6, 215, 1938.
† *H.A.*, 89, 194, 1938.

of +80°, were collected and the mean value for 47 stars at all distances is $-0^m.075$. Both the photovisual and photographic magnitudes are on the international scale. For 16 of the above stars, distances derived spectroscopically, which are reasonably reliable, are available. Grouped according to distance the following results are obtained:—

Dist., ps.	C.I., m.	P.E. of C.I., m.	C.E., m.	No. of Stars
80	−0·06	±0·007	0·00	5
190	− ·08	± ·005	− ·02	9
364	− ·12	± ·011	− ·06	2

The near Ao stars (assumed unobscured) give $-0^m.06$ for the colour index, but the determination is weak. To obtain more material the region between 6^h and 19^h of R.A. and north of Dec. +57½° was examined using Harvard Ao stars only. This region was selected as being more than 20° away from the galactic equator where considerable absorption is known to exist, but colour indices were only used for near stars generally within 200 parsecs. The values of the Harvard photovisual and photographic magnitudes from the mimeographed editions were used, the former requiring the $0^m.10$ zero corrections previously mentioned, while the latter are already on the international scale. The mean colour index for 49 Ao stars brighter than $7^m.5$, at a mean distance of 147 parsecs, is $+0^m.01$ which with the $0^m.10$ zero correction give $-0^m.09$. If stars at distances greater than 100 parsecs are rejected, the result is unchanged. This figure is reasonably near the figure of $-0^m.14$, but only because of the zero correction of $0^m.10$. In the same region of the sky, 21 Ao stars brighter than $6^m.5$, not included above, with a mean distance of 95 parsecs, give a mean colour index of $+0^m.04$, which corrected gives $-0^m.06$.

The results of the separate determinations of the near Ao stars only, when collected, are as follows:—

C.I.		Type	Colour	No. of Stars
m	m			
−0·07	±0·005	Upsala	Mount Wilson	8
− ·05	± ·005	Harv., Upsala, Yerkes	,,	10
− ·10	± ·008	Harv., Yerkes	,,	3
− ·06	± ·007	Mount Wilson	,,	5
− ·06	± ·007	Harvard	,,	5
− ·06	± ·016	,,	*HPv* − *HPg*	21
− ·09	± ·010	,,	,,	49

The mean result is $-0^m.07$. The last two results, it should be noted, have been derived by the application of a zero correction of $-0^m.10$. The probable errors of the colour indices have been calculated from the published probable errors of the colours or magnitudes, and no allowance has been made for the errors in classification of spectral type. It will be observed, however, that a number of determinations of types by several observatories enter into the results, with apparently little effect upon them, and it should be remembered that the stars are mostly brighter than the 8th magnitude so that the error of classification is much smaller than for faint stars. Average errors of one-tenth of a class may be expected, but this could hardly introduce more than $0^m.02$ in the colour index.

It is difficult to understand the difference between the mean value of $-0^m.07$ derived above, and the value of $-0^m.14$ quoted by Seares. Bright Ao stars, within 100 parsecs, yield a figure considerably less blue and it hardly seems likely that they can be suffering absorption. An inspection of the colour indices of the Eros stars, northern stars by Ross and Zug, southern stars by Seares, Sitterly and Joyner, from which Seares derived the very blue value, shows very great ranges. The p.e. of a magnitude from a single plate is $\pm 0^m.09$ and with an average of five plates per star for the northern and four for the southern stars, in both photovisual and photographic magnitudes, the final p.e. of a single colour index is $\pm 0^m.06$. But it is evident from the graph of spectral

type (H.D. Catalogue) against colour index, that there is a large dispersion in the Ao stars, partly due to dispersion in absolute magnitude, partly to accidental error of both type and colour, and partly to space reddening. For it will be observed that in the case of the northern stars the straight mean of the colour indices has been taken, giving $-0^m.15$ for stars with magnitudes varying from 7·4 to 10·0 and therefore at widely different distances. In the case of the southern stars the mean of the colour indices has been taken for 10 faint stars together with 10 bright stars yielding $-0^m.15$, but the separate means for $8^m.9$ and $6^m.5$ are respectively $-0^m.08$ and $-0^m.25$. The group of mean magnitude 8·9 is therefore considerably redder than the group of mean magnitude 6·5, which is composed of stars which presumably cannot be very distant. If we look at the separate values of these 10 stars, six have colour indices excessively blue, far bluer than many B-type stars, three have reasonably normal values and one is typical of an F8 star. The p.e. of the mean colour index of these 20 southern stars ($-0^m.15$) is therefore about $\pm 0^m.04$. For the northern stars, 30 in number, the p.e. of the mean colour index is $\pm 0^m.02$. It would therefore appear that this result, upon which a great deal has been based and which is now becoming widely quoted, has a large probable error, and it would seem that too great an accuracy has been attributed to it.

One of the references quoted by Seares, from which the value of $-0^m.14$ was obtained, was to certain unpublished measures of Stebbins, Huffer and Whitford. The reference is probably to the extensive series of photoelectric colour measures of B- and A-type stars which were subsequently published in 1940. These measures are very accurate, but are measured over a range of wave-lengths only two-thirds of that used in the Mount Wilson colour index determinations. The latter, besides this longer base line, depend on considerably more material. There is also some doubt about the figure used as the coefficient of the colour excess, which is $7E$ and $9E$ for the visual and photographic absorptions respectively. The former has been derived using Wesselink's value of λ 5100 for the effective wave-length of the Harvard visual magnitudes. There is considerable uncertainty in this value of the effective wave-length, for which Brill gives λ 5290. Such a range in the uncertainty of the effective wave-length of a very important and voluminous mass of material, shows the vital necessity of a determination of the wave-lengths used in any work on magnitudes or colours.

The actual measured colour excess in the polar area according to Keenan and Babcock is $+0^m.04$, but this has been multiplied by $7E$ to give a total visual absorption of $0^m.30$. It is evident that the measured quantity is a very small one, and from the previous paragraph it is clear that the coefficient of E by which this small measured reddening has been multiplied is liable to error.

More recently Stebbins, Huffer and Whitford * have published the colours of A-type stars within 10° of the North Pole of the sky, and compared them with the colours of stars within 30° of the north and south galactic poles. The spectra used were by Yerkes, but the Harvard types are also given. If the colours of the stars in the Mount Wilson Catalogue are collected against Harvard types, using the distances as given by the authors, no reddening up to 360 parsecs is revealed. In fact there is a slight effect in the opposite sense, though the weight is not great. If the Mount Wilson colours are used against Yerkes types, there is no evidence of reddening to the same limit of distance. The average difference between the Harvard and Yerkes types is one-tenth of a class, Yerkes classifying later. This has an appreciable effect on the colour excesses, but not sufficient to alter the whole expected effect. Each star was observed once or twice (average one and a half times). The probable error of a colour index is as quoted $\pm 0^m.02$ or slightly less. The Mount Wilson colours are therefore as accurate, if not slightly better determined, than those above. A comparison of the distances given, with those derived by others using the same method of derivation, shows an average uncertainty of 20 per cent. If therefore we look at the graphed results as given by the

* *Ap. J.*, 94, 215, 1941.

authors, bearing in mind the errors in both colours and distances, it will be seen that the slope of reddening for the north pole of rotation may easily be altered. Great weight has been given to the colours at the two greatest distances which, however, are liable to the greatest error. It should be noted that the authors use a value for Ao which when converted to the international scale is $-0^m.15$.

The colour index of Ao stars can be estimated in another way. Spectrophotometric gradients of stars are measured at points free from lines, while in the determination of colour indices the photographic magnitudes integrate all the lines. If therefore we can deduce the effect on the magnitude of integrating all the lines and apply the deduced effect to the value of the colour index (derived from measures of gradient) we should have the colour index of Ao stars. The gradients determined at Greenwich were measured at such points in the blue and red that the final gradient is 2·0 times the colour index. The zero of the gradient system was defined so that the mean gradient of 9 Ao stars was zero. All these stars are within 50 parsecs of the Sun and are therefore free from effects of reddening. The latest list * contains 45 Ao stars. Rejecting 19 of these which are less than 20° from the galactic equator (thereby suffering absorption) or are distant more than 100 parsecs, the remaining 26 stars have a mean relative gradient of $+0^m.04$ or mean relative colour index of $+0^m.02$. Gerasimovič † determined from a microphotometer tracing of the spectrum of α Lyræ (distance 8 parsecs) that the effect of the distortion on the integrated photographic magnitude was $0^m.08$, and stated that the possible upper limit was $0^m.15$. Shajn ‡ has measured the total line and band absorption in the photographic and visual regions for several spectral types. For α Lyræ he deduces a loss of $0^m.09$ in the photographic and a loss of $0^m.01$ in the visual region, thus giving a difference of $0^m.08$ in colour index. Payne-Gaposchkin § remarks, however, that the maximum effect for normal Ao stars is about $0^m.10$. For cAo stars the effect (due mainly to hydrogen lines) will be small, and we may therefore accept $0^m.08$ as the average effect of integration. This, subtracted from $+0^m.02$, gives $-0^m.06$ as the mean colour index of the 26 Ao stars. If the Harvard classification were replaced by that of Mount Wilson the mean of the 9 Ao stars on which the Greenwich gradients depend would be shifted to A1. The effect of this would be to make the colour index of the average Ao stars $0^m.02$ more blue, *i.e.* $-0^m.06 - 0^m.02 = -0^m.08$. If a comparison is made of the Harvard and Bergedorf types for stars of magnitudes similar to those used in the Eros work and in the same portion of the sky, there appears to be on the average no difference.

It therefore appears that the mean result of $-0^m.07$ for bright stars within 150 parsecs from the Sun and probably free from obscuration might be slightly small (numerically), owing to errors of classification, and the effect of variations in the width of Balmer lines and continuous hydrogen absorption. On the other hand, $-0^m.14$ seems unduly large (numerically) and its adoption has exaggerated the reddening near the North Pole of the sky. The total reddening, given as $0^m.10$, is likely to be smaller, and just as the amount has been overestimated, so has the distance been affected, making the effect take place too near. There would appear to be little evidence of reddening at 200 parsecs affecting 8th magnitude stars, and a more reasonable distance would seem to be 500 or 600 parsecs where A-type stars of $9^m.0$ or $10^m.0$ are affected. The early-type stars of the North Polar Sequence would appear to be free of absorption down to photographic magnitude 8·8, *i.e.* numbers 1–9. It is necessary in averaging the colours of A-type stars to be sure that the distances do not cover a great range. To take the mean of the colour indices of a number of A-type stars selected at random, is to falsify the true mean value. A final and more accurate result for the mean colour index of

* *M.N.*, 100, 189, 1940.
† *H.C.*, 339, 1929.
‡ *M.N.*, 94, 647, 1934.
§ *H.A.*, 89, 112, 1935.

unobscured Ao stars can only be derived when further material of better quality is available, and to eliminate the more distant stars it will be necessary to have more determinations of absolute magnitude. The strong concentration of the A-type stars towards the galactic equator, makes the selection of these stars in the region under discussion less easy. It will no longer be sufficient to assume a single absolute magnitude for Ao stars if the distances are to be accurate enough so that their errors are comparable with the errors in colour.

Summary

An account is given of previous work on the region within 10° of the North Pole of the sky, and the Greenwich counts in the region within 3° of the Pole are used in an attempt to locate absorption. Colour excesses from several sources together with spectroscopic distances are used to detect reddening. A discussion of the mean value of the colour index of unobscured Ao stars follows, with a critical examination of the frequently quoted figure of $-0^{m}\cdot14$. It is suggested that the total reddening is smaller than previously accepted and the distance at which it occurs is rather greater. Further work is suggested with certain advisable precautions.

COLOUR TEMPERATURES OF CONVECTIVE STARS

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In the last ten years, observations of stellar colour temperatures have been published by several observatories. In all the published lists the relative gradients G or the absolute gradients ϕ are found to scatter widely for most spectral types.* This colour temperature spread, in terms of the range of ϕ for each spectral type and designated by $p(\phi)$, is deduced from results published by Greenwich † and by Jungfraujoch ‡ observers, and is listed in Table I. Supergiants, peculiar stars, and stars with emission lines in their spectra are entirely excluded from this study.

TABLE I

Colour Temperature Spread

Spectral Type	Greenwich		Jungfraujoch		
	No. of Stars	λλ 6250–4255 $p(\phi_G)$	No. of Stars	λλ 4600–3700 $p(\phi_1)$	λλ 3700–3150 $p(\phi_2)$
Bo	3	0·13	8	0·29 (·15 §)	0·43 (·26 §)
B2	4	·41	5	·09	·10
B3	27	·39 (·31 §)	10	·49	·30
B5	8	·23	6	·39	·46
B8	13	·22	10	·28	·45
Ao	34	·41 (·26 §)	12	·33	·56
A2	24	·34	9	·30	·83
A3	12	·47 (·20 §)	5	·18	·47
A5	19	·40	8	·62	·55
Fo	10	·47	11	·43	·18
F5	5	·30	5	·51	·40
F8	1	...	3	·34	·41
Go	14	·58	3 for ϕ_1 1 for ϕ_2	·55	...
Mean B	55	·28 (·26 §)	39	·31 (·28 §)	·35
,, A	89	·41 (·30 §)	34	·36	·60 ‖
,, F	15	·39	19	·43	·33

* See, for example, Greaves, *M.N.*, 101, 159–160, 1941.

† *M.N.*, 100, 189–195, 1940.

‡ *Ann. d'Ap.*, No. 2, 306–314; No. 4, 412–424, 1938.

§ The smaller values of $p(\phi)$ given inside parentheses are obtained by omitting one star from each of those four types. The stars omitted are ζ Oph (Bo), μ Tau (B3), 29 Cyg (Ao) and ζ^2 Lyr (A3). These stars are much redder (ϕ bigger) than other stars of the same spectral types.

‖ The extremely wide scatter of ϕ_2 for A-type stars may be partly due to some real difference in hydrogen content (the continuous absorption beyond the Balmer series limit being the strongest for A stars). It is also due to the uncertainty of measurement in that region of the spectrum. The observers at Jungfraujoch did remark in the publications that ϕ_2 was not very accurately obtained.

This colour temperature spread cannot be attributed to observational error. In fact, the average probable error of the Greenwich gradients is given as only $\pm 0{\cdot}02$ for the catalogue stars and $\pm 0{\cdot}013$ for the standard stars. The most important factor determining the energy distribution of the continuous spectrum of a star is the coefficient of continuous absorption k. Since only "normal" stars of the main sequence are under consideration, the absolute magnitude effect is quite negligible, and the electron pressures in the outer layers of stars of the same spectral type cannot be far different from one another; therefore those stars of the same type have nearly the same ionization temperature, and quite possibly the same effective temperature. k should then be the same for them unless there is a real difference in chemical composition, especially in hydrogen abundance. The present investigation is centred on another possible cause of the colour temperature spread, namely, convection. It has been realised in recent years that convection plays a more important rôle in the outward transport of energy from the stellar interior than had been suggested by the pioneer work on radiative equilibrium. Stars whose outer layers are in full convective equilibrium do not give the same curve of energy distribution as a black-body of the same effective temperature; therefore for each spectral range and each spectral type the colour temperatures of stars will spread over a range if the stars are convective to varying degrees. This problem was raised by Thackeray *, and the object of this paper is to investigate the effect quantitatively.

Let $I(0, 0)$ denote the intensity of radiation emerging from a star at the centre of the disk, and let I_c, I_R represent the corresponding quantities when the atmosphere is in full convective or radiative equilibrium respectively. Milne † has shown that in radiative equilibrium when k is independent of λ,

$$I_R(0, 0) = 2\pi hc^2\lambda^{-5}\int_0^\infty \frac{e^{-\tau}d\tau}{e^{\alpha'(1+\frac{3}{4}\tau)^{-\frac{1}{3}}} - 1} \quad \text{where} \quad \alpha' = \frac{hc \,.\, 2^{\frac{1}{3}}}{\lambda R T_e}.$$

In convective equilibrium, when again k is independent of λ,

$$I_c(0, 0) = 2\pi hc^2\lambda^{-5}\int_0^\infty \frac{e^{-\tau}d\tau}{e^{\alpha\tau^{-\beta}} - 1},$$

where $\alpha = hc/\lambda R T_e$ and the coefficient β depends not only on γ, the ratio of specific heats, but also on the law of variation of k with pressure and temperature. Here it will be assumed (following Thackeray) that k varies directly with total pressure but is independent of temperature. In these conditions, and with $\theta = 0$ (centre of disk), $\beta = 1/8$ or $1/5$, for $\gamma = 4/3$ or $5/3$ respectively.

In observing radiation from stars we are of course dealing with integrations of $I(\theta)$ over a hemisphere ($\theta = 0$ to $\frac{1}{2}\pi$), but here it has been considered sufficient to confine the comparison of I_R and I_c to the single case $\theta = 0$; differences in limb-darkening will have an effect in the comparison of integrated radiations, which may be expected to be small relative to other uncertainties in the assumed conditions, especially as regards the behaviour of the absorption coefficient k.

The integral in I_c has already been calculated ‡ for $\gamma = 4/3$. In Table II, values for $\gamma = 5/3$, also obtained by numerical integration, are given for $\alpha = 0{\cdot}7175$ to $10{\cdot}0$.

In Table III I_c and I_R are directly compared at various wave-lengths and temperatures. § For $\gamma = 4/3$ it will be noticed that the differences are relatively small; moreover, I_c tends to exceed I_R around the wave-length of maximum intensity while the inequality changes sign at the extremes of short or long wave-length. For $\gamma = 5/3$ the differences are considerably larger. The differences tend to diminish towards higher

* *M.N.*, 100, 614, 1940. † *M.N.*, 81, 383, 1921. ‡ Thackeray, *op. cit.*

§ I_R was computed by interpolating in Milne's tables of $f(\alpha, p)$ with $p = 1{\cdot}5$ (*Phil. Trans.*, A, 223, 209, 1922).

TABLE II

Values of $-\log I(\alpha)$ *for* $\gamma = 5/3$

$$\left(I(\alpha) = \int_0^\infty \frac{e^{-\tau} d\tau}{e^{\alpha\tau^{-1/5}} - 1} \right)$$

α	$-\log_{10} I(\alpha)$	α	$-\log_{10} I(\alpha)$	α	$-\log_{10} I(\alpha)$
0·7175	0·07172	3·5	1·571	7·0	2·9960
0·884	0·2062	4·0	1·7837	7·5	3·191
1·0	0·2907	4·5	1·989	8·0	3·386
1·5	0·6039	5·0	2·194	8·5	3·576
2·0	0·8723	5·5	2·397	9·0	3·766
2·5	1·1169	6·0	2·597	9·5	3·951
3·0	1·3475	6·5	2·797	10·0	4·1344

temperatures, but we cannot be sure from the present calculations that this would continue to hold good in the regions of short wave-length ($\lambda < 3000$) near the intensity maximum.

TABLE III

	$\log_{10}(I_c/I_R)$							$\Delta\phi$		
T_e \ λ	·315	·37	·4255	·46	·53	·625	·77	·315–·37	·37–·46	·425–·625
15000°	*−0·010*	*−·001*	*+·001*	*−·001*	*−·008*	...	...			
	0·0	+·005	+·003	·000	−·009	...	...			
10000	*+0·051*	*+·038*	*+·025*	*+·014*	*+·004*	*−·002*	...	*−·063*	*−·104*	*−·083*
	+0·014	+·011	+·005	·000	−·004	−·004	...	−·015	−·048	−·028
6000	*+0·118*	*+·097*	*+·076*	*+·065*	*+·050*	*+·035*	*+·014*	*−·102*	*−·139*	*−·126*
	−0·001	+·003	+·008	+·010	+·014	+·010	·000	+·020	+·030	+·006
3000	*+0·357*	*+·281*	*+·220*	*+·194*	*+·158*	*+·126*	*+·089*	*−·371*	*−·379*	*−·288*
	−0·075	−·051	−·040	−·022	−·009	·000	+·003	+·117	+·126	+·123

Upper figures (*italics*): $\gamma = 5/3$.
Lower figures (roman): $\gamma = 4/3$.
Unit of wave-length, *microns*.

It is of interest to investigate the variation of $\log (I_c/I_R)$ with $1/\lambda$. If the variation were *linear* this would imply an apparent difference in colour temperature between convective and radiative stars at the same effective temperature and that mere extension of the observed range of wave-length would not help to distinguish between the effects of convection and of a real temperature-difference. In fact, the variation is not linear, but in order to study the importance of the influence of convection upon colour, the differences in gradient ϕ of convective and radiative stars at the same effective temperature have been calculated from the relation

$$\Delta\phi = -2{\cdot}303(A_{\lambda_1} - A_{\lambda_2}) \Big/ \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right),$$

where $A = \log (I_c/I_R)$ and λ is expressed in microns. Table III shows the results for the three wave-length ranges 6250 – 4255, 4600 – 3700, 3700 – 3150 A., the first being

the range for the colour temperatures observed at Greenwich (also Göttingen and Ann Arbor), the other two being the ranges for which Barbier, Chalonge and their colleagues obtained colour temperatures at Jungfraujoch (λ 4255 is in fact the harmonic mean of eight points in the blue chosen by Greenwich observers, whereas λ 6250 is that of eight points in the red).

Comparison with the ranges $p(\phi)$ in observed colour temperatures of B to F stars (Table I) shows that the effect of convection upon colour temperatures higher than 6000° is negligible. Since the hydrogen zone of convection only emerges near to the optical surfaces of stars for these higher temperatures it might be concluded that in no circumstances could convection appreciably influence observed colours. However, this investigation indicates that if convection currents exist to an appreciable extent in the photospheres of stars of type G and later (and spectroscopically there appears to be good evidence for such currents), then they may have an important influence on the distribution of energy in the continuous spectrum. The onset of convection demands a small value of the effective ratio of specific heats, and presumably we must look to some cause other than the ionisation of hydrogen (*e.g.* the presence of molecular compounds) for an explanation applicable to the cooler stars. A point of interest arising from Table III is the reversal in sign of $\Delta\phi$ for $\gamma = 5/3$ and $4/3$ at $T = 3000$ and 6000. It appears that in M-type stars convection should make a star appear redder or bluer according as $\gamma = 4/3$ or $5/3$ respectively. It is, however, doubtful whether a value of γ making $\Delta\phi$ negative for $T = 3000$ to 6000 would not be so high as to damp out convection altogether. Thus it may be concluded from these calculations that appreciable colour effects due to convection should be expected primarily in a slight reddening of stars of late type.

It must be remembered that the above calculation of I_c involves the assumption that k is proportional to p and independent of T. Although this seems to agree with conditions in the solar atmosphere it may not hold for stars of other types. For this reason the numerical results of this investigation should be used with caution at least as far as comparison with observation is concerned. It may be pointed out that an increase in the sensitivity of k to pressure or temperature (k increasing in the same sense) will decrease the coefficient β and thus produce the same effect on colour temperatures of convective stars as a diminution of γ.

A further point to be borne in mind is that the problem has been idealised here in that the comparison has been restricted to that of a convective and a radiative star identical in other respects; whereas such a radical difference in physical states of two atmospheres must almost certainly imply other differences (such as chemical composition and consequently the behaviour of k) which would also influence the colour-distribution of the emergent radiation.

Summary

The wave-length distribution of emergent radiation at the centre of the disk from atmospheres in full radiative and convective equilibrium is compared. For temperatures higher than 6000° the influence of convection upon colour temperature is too small to contribute appreciably to the observed spread in colour temperature of early-type stars. Among stars of late type convection can produce an appreciable reddening.

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CONVECTIVE EQUILIBRIUM AND SOLAR LIMB DARKENING—A CORRECTION

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While reading the first version of the preceding paper, the writer detected an error in his original paper * on the subject, which has necessitated revision of the results given both in that paper and in Dr. Tai's. The error lay in the evaluation of the factor f which normalises the emergent radiation to that given out by a black-body of temperature T_e. In this evaluation a factor 2 was inadvertently omitted whereby f was estimated too large by a factor $2^{\frac{1}{4}}$. In other words, all the results given for the disk correspond to an effective temperature 6826° instead of 5740°, and for the spot to $T_e = 5114°$ instead of 4300°.

In order to clarify the position it will be as well to record the corrected equations for the most general case of an absorption coefficient varying with total pressure p and temperature T according to the law

$$k \propto p^a T^b.$$

For an atmosphere in fully convective equilibrium ($p \propto \rho^\gamma$) the relation between T and optical depth τ is then given by

$$T = f \,.\, \tau^\beta,$$

where f depends only on T_e and γ, and

$$\frac{1}{\beta} = b + \frac{\gamma}{\gamma - 1}(a + 1).$$

The total emergent radiation at angle θ to the normal has energy given by

$$T_\theta^4 = \int_0^\infty f^4 (\tau \cos\theta)^{4\beta} e^{-\tau} d\tau = f^4 (\cos\theta)^4 \Gamma(4\beta + 1).$$

This must now be integrated over a hemisphere to compare with the effective temperature of the radiation from the whole visible disk.

Thus

$$T_e^4 = f^4 \Gamma(4\beta + 1) \int_0^{\pi/2} (\cos\theta)^4 \, 2 \sin\theta \,.\, \cos\theta \,.\, d\theta$$
$$= f^4 \frac{\Gamma(4\beta + 1)}{2\beta + 1}.$$

In the particular case considered, $a = 1$, $b = 0$, we have

for $\gamma = \frac{4}{3}$: $\beta = \frac{1}{8}$ $T_e^4 = \frac{4}{5} f^4 \Gamma(1{\cdot}5)$,

or for $\gamma = \frac{5}{3}$: $\beta = \frac{1}{5}$ $T_e^4 = \frac{5}{7} f^4 \Gamma(1{\cdot}8)$.

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* *M.N.*, 100, 614, 1940. Hereinafter referred to as paper I.

Thus with $\gamma = \frac{4}{3}$ and $T_e = 5740$, $f = 6256$ instead of 7439 as given in paper I. It is to be noted that f is the value of T at $\tau = 1$.

This change has no effect on the predicted darkening for integrated radiation (fig. 1*a* of I). But the darkening at individual wave-lengths and the distribution of energy with wave-length are of course both affected through the coefficient $\alpha\left(=\frac{hc}{R\lambda f(\cos\theta)^{\frac{1}{3}}}\right)$. Table I shows the recomputed wave-length distribution of the emergent radiation at the centre of the disk for $T_e = 5740$. The wave-length of maximum intensity was investigated in detail and found to be 4600 ± 10 A. This agrees within the computational error with the λ_{max} obtained by substituting $T_e = 5740$ in Milne's * similar computations for an atmosphere in complete radiative equilibrium. As Milne pointed out, this maximum is shifted by 4·3 per cent. to the violet from the position for a black-body.

TABLE I

Wave-length Distribution of Radiation from Centre of Disk

Convective Equilibrium, $\gamma = \frac{4}{3}$. $\quad I_c = \frac{2\pi hc^2}{\lambda^5}\int_0 \frac{e^{-\tau}d\tau}{e^{a\tau^{-1/3}}-1}$

λ (1000 A.)	$I_c \times 10^{-14}$	λ (1000 A.)	$I_c \times 10^{-14}$	λ (1000 A.)	$I_c \times 10^{-14}$
3·0	6·71	4·8	10·67	6·5	8·22
3·5	8·94	5·0	10·53	7·0	7·38
3·7	9·56	5·2	10·34	7·5	6·61
4·0	10·23	5·4	10·06	8·0	5·87
4·2	10·55	5·6	9·78	9·0	4·60
4·4	10·66	5·8	9·42	10·0	3·63
4·6	10·72	6·0	9·09		

In Table II (*a*) the recomputed limb-darkening is given for the same wave-lengths as in Table II of paper I, while in Table II (*b*) the darkening is compared with Rauden-busch's observations at wave-lengths 4260, 5060, 5960, 6700 A.

TABLE II (*a*)

Limb-darkening (*cf.* Table II of Paper I)

Unit of intensity = 100 at $\sin\theta = 0{\cdot}0$.

	$\lambda = 3220$			$\lambda = 4500$			$\lambda = 6000$			$\lambda = 8500$			$\lambda = 15000$		
$\sin\theta$	I_c	I_0	I_R	I_c	I_0	I_R	I_c	I_0	I_R	I_c	I_0	I_R	I_c		I_R
0·7	74	72	70	80	82	76	85	86	84	88	90	88	92	95	92
0·85	57	56	52	66	68	65	73	76	74	79	81	79	85	90	85
0·95	34	38	36	45	49	50	55	61	61	63	70	68	74	82	78
0·98	22	29		32	35		43	47		52	57		65	69	
0·99	15			25	24		35			45			59	50	

* *M.N.*, 81, 384, 1921.

TABLE II (*b*)

Limb-darkening for Convective Equilibrium compared with Raudenbusch (A.N., 266, 301, 1938)

	$\lambda = 4260$			$\lambda = 5060$			$\lambda = 5960$			$\lambda = 6700$		
sec θ	I_c	I_0	I_R'	I_c	I_0	I_R'	I_c	I_0	I_R'	I_c	I_0	I_R'
1·25	859	855	851	879	871	876	897	895	895	906	910	906
1·50	757	753	753	791	785	791	818	825	821	834	847	840
2·00	615	626	631	662	668	684	706	729	728	730	758	755
2·50	522	548	559	575	592	620	627	667	670	656	706	702
3·00	457	491	512	513	541	577	568	623	631	600	668	665

Unit of intensity = 1000 at sec $\theta = 1{\cdot}0$.

I_c : Intensity computed for convective equilibrium ($\gamma = \frac{4}{3}$).
I_0 : Observed intensity.
I_R' : Intensity computed for radiative equilibrium, with blanketing effect.

The comparison Spot-Disk of paper I is comparatively unaffected by the erroneous value of f and it is not proposed to include detailed results here. While the actual values of S_c/D_c (depending primarily on the assumed spot-temperature) are somewhat diminished, the variation with θ, which forms the chief basis of comparison with observation, is changed by less than 3 per cent. from sin $\theta = 0$ to 0·95. The wave-length variation is affected by an error of the same order. The variation of S_c/D_R is at variance with observation as before. Thus the conclusion stands that the observations may be equally well represented by spot and disk *both* in radiative equilibrium, or *both* in convective equilibrium.

PARTIAL SOLAR ECLIPSE 1942 SEPTEMBER 10

Royal Observatory, Greenwich

(Communicated by the Astronomer Royal)

A series of six photographs was taken just after the beginning of the partial solar eclipse on 1942 September 10 with the 4-inch photoheliograph at the Royal Observatory, Greenwich. The first plate was exposed 4 seconds after the revised predicted time, given by the Astronomer Royal as $15^h\ 24^m\ 14^s{\cdot}5$ U.T. The first five plates were exposed at intervals of 12 seconds, the sixth at $15^h\ 25^m\ 26^s{\cdot}5$. The fifth plate was spoilt by an accidental double exposure. The definition on all the plates was satisfactory.

The angle 2θ, subtended by the cusp at the centre of the Sun's image, was measured on each plate with the solar micrometer used for sunspot positions.

The positions of the Sun and Moon were corrected for parallax and the augmentation of the Moon's angular diameter was computed. As the changes in parallactic displacement are not linear in time, the rates of change with time of the differences in the Sun's and Moon's R.A. and Dec. were computed for an instant near the expected time of first contact.

The computed distance between the centres of the Sun and Moon was found to be

$$D = 1885''{\cdot}8 - 18''{\cdot}898t + 0''{\cdot}07272t^2,$$

where t is measured in minutes from the instant of first contact and $1885''{\cdot}8$ is the sum of the augmented semi-diameters of Sun and Moon.

This can be put in the form

$$2\left(R_\odot \sin^2\frac{\theta}{2} + R_\leftmoon \sin^2\frac{\phi}{2}\right) = 18''{\cdot}898t - 0''{\cdot}07272t^2,$$

where 2ϕ is the angle subtended by the cusp at the centre of the Moon. ϕ is obtained from the measured values of θ by means of

$$R_\odot \sin\theta = R_\leftmoon \sin\phi.$$

For each value of θ, the corresponding values of t can be inferred. The times of exposure, measured values of θ, deduced intervals from the instant of first contact (in seconds) and the resulting times of first contact are given in the following table:—

Time of Photograph U.T.	θ	t	Concluded Time of First Contact U.T.
h m s	° ′	s	h m s
15 24 18·5	1 41·5	2·67	15 24 15·83
24 30·5	4 12·5	16·51	13·99
24 42·5	5 36·5	29·35	13·15
24 54·5	6 33·0	40·05	14·45
25 26·5	8 41·0	70·45	16·05

The concluded time of first contact is

$$15^h\ 24^m\ 14^s{\cdot}69 \pm 0^s{\cdot}37 \text{ U.T.}$$

The photographs were taken and measured by Messrs. H. Barton and P. Laurie. The eclipse was seen to have begun at $15^h\ 24^m\ 25^s{\cdot}7$ by Mr. E. G. Martin, who observed with the 7-inch Sheepshanks Equatorial, power 140.

HORNSBY'S OBSERVATIONS OF JUPITER, 1774–1784

Bernard Francis Bawtree

(Communicated by H. Spencer Jones)

(Received 1942 November 23)

The following are comparisons of Hornsby's observations of Jupiter for 1774–1784 with the places obtained from Hill's tables in *Astronomical Papers of the American Ephemeris*, vol. VII, part I, 1895.

Hornsby's observations were published in *The Observations of the Reverend Thomas Hornsby*, Oxford, 1932.

Jupiter

Observed *minus* Computed

	R.A.	Dec.
1774	s	″
Jan. 31	−0·35	+1·7
Feb. 2	−0·05	−3·1
5	−0·08	−5·0
20	−0·22	−2·3
Mar. 14	−0·02	−1·1
Apr. 30	−0·05	+1·6
May 14	−0·15	−2·1
June 6	−0·06	−1·8
July 22	−0·03	+1·6
24	+0·02	+1·5
Nov. 13	−0·14	+1·9
28	−0·23	−1·8
Dec. 30	−0·11	−0·4
1775		
Jan. 1	−0·14	+3·4
6	−0·27	+1·7
13	−0·08	−1·6
17	−0·19	−3·0
24	−0·27	−1·3
Apr. 16	−0·38	+1·9
19	−0·17	+1·6
20	−0·27	+0·2
May 2	−0·40	+0·2
3	−0·35	−0·3
4	−0·25	0·0
June 4	−0·09	+0·3
16	−0·33	+1·1
25	−0·10	+4·0
July 19	−0·10	+2·1
22	−0·16	0·0
28	−0·06	−0·3
Aug. 13	−0·21	+2·0
24	−0·09	+0·6
27	−0·09	−1·8
28	−0·08	+0·4
29	−0·09	+2·1
Sept. 11	−0·15	−0·5
21	−0·07	−0·1
29	−0·13	+4·3
Dec. 6	−0·41	+1·6
8	−0·26	+1·4
9	−0·45	+2·4
1776		
Jan. 3	−0·23	−0·7
26	−0·35	+3·6
31	−0·32	+1·3
Feb. 7	−0·11	+3·1
8	−0·16	+1·6
11	−0·05	−0·2
16	−0·18	−0·4
Mar. 11	−0·14	+0·9
12	−0·08	+0·9
17	−0·11	−2·3
26	−0·04	−2·3
29	+0·06	+0·7
Apr. 10	−0·09	−0·4
21	−0·36	+0·5
28	−0·07	+3·1
May 13	−0·08	−0·9
28	−0·12	+2·1
June 2	−0·25	+1·8
14	−0·36	−2·6
July 17	−0·12	+0·1
22	−0·05	+6·2
24	−0·18	−0·6
25	−0·04	+1·3
29	−0·15	−0·7
31	−0·04	+1·7
Aug. 1	−0·03	+3·0
3	−0·18	−1·8
16	−0·26	−2·2
19	−0·15	0·0
21	−0·24	+2·1
22	−0·14	−2·2
31	−0·05	+1·1
Sept. 5	+0·05	+1·6
7	−0·13	+2·7
18	+0·14	+0·5
20	−0·14	+1·5
21	−0·07	+0·7
23	+0·03	+3·0
27	−0·05	+1·2
28	+0·09	+4·4
1777		
Jan. 7	−0·10	+2·0
16	−0·26	+0·8
1777		
Mar. 24	+0·01	+4·5
Apr. 13	+0·03	−1·1
16	−0·09	+4·7
17	+0·03	+0·7
25	+0·01	+0·1
26	+0·05	−0·5
27	−0·01	+0·7
May 2	−0·04	+1·0
4	+0·01	+0·6
5	−0·16	+0·1
7	−0·19	−1·9
17	−0·03	+3·0
26	−0·11	+0·1
July 2	−0·11	+8·2
17	−0·35	−0·4
Sept. 13	+0·10	+1·1
14	+0·09	+1·8
15	+0·10	+2·8
16	+0·07	+1·5
22	−0·05	+3·3
25	+0·08	+1·4
Oct. 1	+0·13	−0·5
4	−0·05	+1·9
6	+0·06	−0·6
18	+0·11	+0·4
19	+0·09	−1·9
20	+0·06	−1·6
Nov. 4	−0·06	−0·6
6	+0·12	−0·2
19	−0·08	−1·2
1778		
Feb. 9	−0·01	+1·0
Mar. 18	0·00	+1·6
25	+0·04	+4·5
26	−0·24	+0·1
27	−0·03	+0·5
Apr. 5	−0·08	+1·3
11	−0·18	+0·9
12	+0·07	−0·6
15	−0·03	+4·0
18	+0·03	+2·4
19	−0·01	+2·5
26	+0·08	−2·2

Jupiter—(continued)

	R.A.	Dec.
1778	s	″
May 8	−0·14	+1·9
9	−0·08	−0·4
12	−0·11	−1·8
14	−0·02	+1·3
19	−0·06	+0·3
23	−0·04	+1·8
24	−0·09	+2·1
29	+0·02	−1·2
June 3	−0·04	−0·8
12	+0·08	0·0
20	−0·04	−0·7
28	−0·02	+0·6
July 6	−0·16	−3·2
8	−0·10	−0·1
14	−0·19	−3·8
15	−0·12	−1·2
16	−0·15	−1·4
Aug. 9	−0·21	−0·5
Sept. 19	+0·04	−0·4
30	−0·19	−1·7
Oct. 10	+0·06	+5·0
12	−0·26	−0·3
13	+0·16	+1·8
14	−0·03	−2·8
15	−0·07	−1·9
16	+0·14	+1·1
17	+0·04	−1·6
20	−0·17	−0·9
25	−0·05	+0·3
31	+0·22	+0·2
Nov. 11	−0·04	+3·0
12	+0·08	−2·4
24	+0·05	−2·6
Dec. 5	+0·10	+0·8
13	+0·13	−0·7
14	+0·21	−2·2
19	+0·06	−0·8
1779		
Mar. 11	−0·18	+4·6
12	−0·10	+0·4
13	−0·03	+1·7
26	−0·15	+1·4
30	+0·02	+1·8
31	−0·09	+1·7
Apr. 1	−0·18	+5·3
28	+0·24	+5·7
May 6	+0·09	+4·1
19	−0·09	−0·4
23	+0·04	−1·6
27	+0·12	−2·2
31	+0·06	+0·9
June 6	−0·06	−2·7
15	+0·06	+2·2
17	−0·05	+0·6
19	+0·05	+0·4
20	+0·11	−1·6
23	−0·03	−1·1
26	+0·06	−0·9
28	+0·15	−1·8

	R.A.	Dec.
1779	s	″
Sept. 24	+0·07	−6·4
Oct. 19	−0·03	0·0
21	0·00	−1·8
Nov. 18	+0·02	+1·2
29	−0·07	−0·8
Dec. 3	+0·04	−5·0
4	−0·01	+0·9
5	+0·15	+1·3
21	+0·19	−4·1
1780		
Jan. 13	+0·19	+2·8
Apr. 13	+0·14	−0·5
29	+0·04	+0·2
May 15	−0·01	−1·1
17	+0·01	−1·0
22	+0·05	−1·0
27	+0·12	−1·1
28	+0·10	−0·1
29	+0·04	−1·4
June 15	−0·09	−0·2
19	+0·12	+0·2
21	−0·07	+7·0
23	0·00	−2·2
24	0·00	−1·6
25	+0·04	+0·7
29	+0·02	−1·8
30	−0·05	−1·5
July 23	−0·28	−1·8
25	+0·07	−3·0
26	−0·02	−1·5
27	+0·10	0·0
28	−0·01	−3·6
29	0·00	−1·9
31	−0·06	+0·1
Aug. 2	−0·12	−3·3
4	+0·04	0·0
6	+0·01	+2·6
7	+0·03	+0·9
9	−0·02	−0·4
10	−0·04	−1·8
11	+0·06	−1·5
13	+0·01	+1·6
21	+0·04	+1·4
23	−0·13	+3·3
24	+0·15	+2·7
26	−0·11	+2·4
27	+0·15	+1·4
28	−0·05	+2·7
Sept. 24	+0·14	−1·1
Nov. 18	+0·01	−1·7
Dec. 18	−0·06	−3·5
1781		
Jan. 26	+0·24	−2·9
30	+0·38	+0·9
Feb. 6	+0·09	−3·2
15	+0·13	+0·6
17	+0·17	−1·5
May 11	+0·02	−0·3

	R.A.	Dec.
1781	s	″
May 12	+0·14	+0·4
26	+0·16	+1·9
27	+0·15	+0·4
28	+0·18	−1·9
31	+0·14	+1·8
June 1	+0·07	+2·1
6	+9·04 *	+3·4
13	0·00	−2·7
July 9	+0·07	−0·5
17	−0·05	+0·3
18	+0·05	+1·0
30	+0·06	−0·3
31	−0·07	−1·4
Aug. 13	−0·11	0·0
19	−0·24	+0·9
Sept. 1	−0·09	−0·9
9	−0·06	−2·0
16	−0·11	+2·1
1782		
Feb. 27	+0·38	−0·5
Mar. 12	+0·35	−1·0
15	+0·26	+1·9
17	+0·14	+4·9
23	+0·18	+5·1
June 14	−0·07	+5·9
15	−0·06	+1·3
19	+0·21	+7·0
1783		
Jan. 22	+0·01	−2·2
Mar. 7	+0·10	+0·8
15	+0·22	+0·3
16	+0·29	−1·1
17	+0·32	−0·7
21	+0·17	−1·3
Apr. 2	+0·33	+1·7
5	+0·36	+1·1
6	+0·31	+1·8
7	+0·22	+2·4
18	−0·40	+1·9
Sept. 9	+0·18	−4·6
23	−0·09	+2·8
27	+0·09	−0·7
28	−0·01	+1·4
29	−0·15	−2·5
Oct. 14	+0·09	+1·7
18	+0·08	+0·8
21	+0·21	−2·8
Nov. 4	+0·13	−1·9
1784		
Oct. 14	+0·22	+0·1
15	+0·13	−2·0
16	+0·02	−0·3
26	+0·05	−1·8
27	+0·02	−1·9
28	+0·09	−3·6
Nov. 12	+0·19	+3·4
17	+0·07	−1·9
21	+0·12	−0·9
Dec. 15	0·00	−1·1

* The following note occurs in Hornsby's *Meridian Observations*, page 128: "The observed R.A. appears to be about 9^s too great."

LIST OF ADDITIONS

TO THE

LIBRARY OF THE SOCIETY

1942 FEBRUARY 1–AUGUST 31

Periodicals received by exchange are not included in this list.

Unless otherwise indicated, the donor is the author or the Institution concerned.

An asterisk (*) indicates that the work is an excerpt.
A dagger (†) indicates that the work has been purchased.

Bawtree (Bernard F.):

Sun Tables for 0 to 2000 A.D. 4to. St. Albans, 1941

Burns (P. F.):

First Steps in Astronomy. 8vo. London, 1942
(*Messrs. Ginn & Co., Ltd.*)

Coccaeus (Jacobus):

Epistola de mundi quae circumferuntur systematis et nova alio illis certiore dialogismum paradoxum complexa. 4to. Amstelaedami, 1660
(*Mr. G. Wyatt.*)

Debenham (Frank):

Astrographics. First Steps in Navigation by the Stars. Second Edition. 8vo. Cambridge, 1942

de Bruhl (Count):

Three registers of a pocket chronometer and the observations from which they were collected by . . ., with an Account from Vice-Admiral Campbell, of the First of Mr. Mudge's Time-Keepers, in a Voyage to and from Newfoundland. 4to. s.l. s.d.
(*Major George Anne.*)

Endeavour: A quarterly review designed to record the progress of the sciences in the service of mankind. Vol. 1, Nos. 1–3. 4to. London, 1942
(*Imperial Chemical Industries.*)

H.E. (Sir H. C. Englefield):

Description of a method of taking the differences of Right Ascension and Declination with the Reticule Rhomboide of Dr. Bradley. Without placing the instrument in the plane of the equator. 4to. Bath, *c.* 1785
(*Major George Anne.*)

Euler (Leonhard):

†Opera omnia; sub auspiciis Societatis Scientiarum naturalium Helveticae edenda curaverunt A. Speiser, L. G. du Pasquier, H. Brandt. Series I. Opera Mathematica. Vol. 4.

Vol. 4. Leonhardi Euleri commentationes Arithmeticae volumen tertium edidit Rudolf Fueter. 4to. Genevae, 1941

Gamow (George):

†The Birth and Death of the Sun. 8vo. New York, 1940

Jeffreys (Harold):

*Some Application of the Method of Minimum χ'^2. 4to. Cambridge, 1941

Jones (V. C.):

*Model Planetarium. 8vo. s.l. 1941

Merrill (Paul W.):

*Spectra of Long-Period Variable Stars. 8vo. Chicago, 1940

Mexico, Sociedad Astronomica de Mexico:

El Universo. Año 1, No. 3. 8vo. Mexico, 1942

Minneapolis, University of Minnesota :

Bruce Proper Motion Survey. The General Catalogue. By W. J. Luyten. 4to. Minneapolis, 1941

A. The Proper Motions of 1465 Stars South of Declination $-80°$.

B. The Proper Motions of 14500 Stars between Declinations $-60°\ 00''$ and $-79°\ 59'$.

Section 1. BPM 1501–6000. $0^h\ 0^m.00 - 9^h\ 47^m.9$.

2. 6001–11000. $9^h\ 47^m.9 - 17^h\ 43^m.8$.

3. 11000–16000. $17^h\ 43^m.8 - 23^h\ 59^m.9$.

C. The Proper Motions of 12540 Stars between Declinations $-50°\ 00'$ and $-59°\ 59'$.

Section 1. BPM 16001–22000. $0^h\ 00^m.0 - 13^h\ 22^m.1$.

2. 22001–28535. $13^h\ 22^m.1 - 23^h\ 59^m.8$.

New York, Federal Works Agency : Works Projects Administration :

†Works Projects Administration Tables. Prepared by the Federal Works Agency and sponsored by the National Bureau of Standards. Vols. 1–10, 12. fol. & 4to. New York, 1939–41

Vol. 1. Tables of the First Ten Powers of the Integers from 1 to 1000.

2. Tables of the Exponential Function e^x.

3. Tables of the Circular and Hyperbolic Sines and Cosines for Radian Arguments.

4. Tables of Sines and Cosines for Radian Arguments.

5. Tables of Sine, Cosine and Exponential Integrals. I.

6. Ditto. II.

7. Tables of Natural Logarithms. I.

8. Ditto. II.

9. Ditto. III.

10. Ditto. IV.

12. Tables of Probability Functions. I.

Piggott (Edward) :

Letters, etc., various. MSS.
(*Major George Anne.*)

Piggott (Nathanial) :

Letters, etc., various. MSS.
(*Major George Anne.*)

Pringle (Sir John) :

A Discourse on the Invention and Improvements of the Reflecting Telescope. Delivered at the Anniversary Meeting of the Royal Society, November 30, 1777. 4to. London, 1778
(*Major George Anne.*)

Senex (J.) :

Zodiacus Stellatus fixas omnes hactenus cognitas, ad quas lunae appulsus ullibi terrarum telescopio observari poterunt, complexus. Delineavit et sculpsit Johan. Senex.
(*Major George Anne.*) Long roll on linen

Toaldo (Guiseppe) :

De methodo longitudinum ex observato lunae transitu per meridianum ad celeberrimum virum Dr. Nevil Maskelyne astronomum regium Grenovicensem amplissimi longitudinum collegii Britannici Membrum, &c., &c. Epistola Josephi Toaldi Astron. Prof. Patavini Soc. R. Sodalis &c. 4to. Patavii, 1784
(*Major George Anne.*)

Tucumán, Universidad Nacional :

Revista. Serie A. Matematicas y Fisica teorica. Volumen 2, Nos. 1 y 2. Diciembre de 1941. Cr. 8vo. Tucumán, 1941
(*Messrs. J. M. Dent & Sons, Ltd.*)

Tweedale (C. L.) :

Reflecting Telescope Making. 8vo. London, 1942

Washington, National Geographic Society :

Contributed Technical Papers. Solar Eclipse Series. No. 2. 8vo. Washington, 1942
No 2. National Bureau of Standards Solar Eclipse Expedition of 1940 to Brazil.

ERRATUM

M.N., 101, No. 3, List of Additions to the Library:

Berthoud (F.) :

—: Traité d'Horlogerie. 4to. Paris, 1755

should read

Lepaute (J. A.) :

Traité d'Horlogerie. 4to. Paris, 1755

LIST OF ADDITIONS

TO THE

LIBRARY OF THE SOCIETY

1942 SEPTEMBER 1 TO DECEMBER 31

Periodicals received by exchange are not included in this list.

Unless otherwise indicated, the donor is the author or the Institution concerned.

An asterisk (*) indicates that the work is an excerpt.
A dagger (†) indicates that the work has been purchased.

Abastuman, Astrophysical Observatory of the Academy of Sciences of the Georgian S.S.R.

Bulletin, No. 6 — 8vo. Abastumani, 1942

Association of Special Libraries and Information Bureaux:

War-time guides to British sources of specialised information. No. 4. — fol. typescript, 1942

No. 4. Scientific and Technical Periodicals received from the U.S.S.R. Compiled at the request of the British Council on the recommendation of the Anglo-Soviet Scientific Collaboration Committee. December 1942.

Chandrasekhar (S.):

†Principles of Stellar Dynamics. (Astrophysical Monographs sponsored by the *Astrophysical Journal*.) — 8vo. Chicago, 1942

Committee for the Distribution of Astronomical Literature:

Monthly Astronomical News-Letter. No. 1 and 3. — 4to. typescript, 1942

Córdoba, Observatorio Nacional Argentino:

Resultados, Vol. 37. — 4to. Córdoba, 1941

Vol. 37. Catálogo fundamental general.

Fries (R. E.):

On the Origin and Foundation of the Royal Swedish Academy of Science. A speech delivered at the festival meeting celebrating the bicentenary of the Academy on the 11th of October 1939.
(*Royal Swedish Academy of Science.*) — 8vo. Stockholm, 1940

Gamow (George):

†Biography of the Earth. Its Past, Present and Future. — 8vo. New York, 1941

Hase (V.):

*Peculiar changes in the shell spectrum of γ Cassiopeiæ. — 8vo. Abastumani, 1941

H.M. Nautical Almanac Office:

Apparent Places of Fundamental Stars, 1943. Containing the 1535 stars in the Third Fundamental Catalogue (FK3) of the Berliner Jahrbuch. Produced by international co-operation under the auspices of the International Astronomical Union.
(*H.M. Stationery Office.*) — Crn. 8vo. London, 1942

Jeans (Sir James):

*Our Home in Space. 4to. London, 1942
(*British Broadcasting Corporation.*)

Jones (H. Spencer):

*John Stanley Plaskett, 1865–1941. Crn. 8vo. London, 1942

London, The Physical Society:

†Reports on Progress in Physics. Volumes VI, VII, VIII. 8vo. London, 1940–1942

Mees (C. E. Kenneth):

The Theory of the Photographic Process. 8vo. New York, 1942

New York, Federal Works Agency: Works Projects Administration:

†Works Projects Administration Tables. Prepared by the Federal Works Agency and sponsored by the National Bureau of Standards. Vols. 11 and 13. 4to. New York, 1942

Vol. 11. Tables of Sine and Cosine integrals for arguments from 10 to 100.
Vol. 13. Tables of Probability Functions. II.

Nordenmark (N. V. E.):

*Nagra Aterfunna Wargentin-Manuskript. 8vo. Stockholm, 1940

Raxo (Francisco Fernandez):

†De Cometis, et prodigiosis eorum portentis, libri quatuor. 4to. Madriti, 1578

Stockholm, Royal Swedish Academy of Science:

Bicentenaire de l'Académie Royale des Sciences de Suède, Année 1939. Rapport élaboré sur la demande du Comité du Jubilé par Arne Holmberg. 8vo. Stockholm, 1941

Sydney Observatory:

Astrographic Catalogue, 1900·0. Vol. XXIII. 4to. Sydney, 1942

Vol. XXIII. R.A. 12^h to 18^h. Dec. $-56°$ to $-58°$.

Tilney (W. A.):

†Marching or Flying by Night without a compass with Time Table of directional stars. Compiled for latitudes 40° N.–55° N. Eleventh Edition. Revised.
16mo. London, 1940

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3. Diagrams should be drawn on a scale from two to three times the intended final size, and should be in a state suitable for direct photographic reproduction. *All lettering should be inserted in pencil.* In the event of diagrams being unsuitably drawn the Secretaries are empowered to request authors to have them re-drawn, at their own expense, by a professional artist. The above instructions do not, of course, apply to half-tone blocks.

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